Problems

Problems marked with * are meant to be solved without the aid of a computer.

(1) * Suppose that you wish to fit a linear function, $y = at$, to a set of data, d_i, $i = 1, 2, \ldots, N$, taken at values of the independent variable, t_i, $i = 1, 2, \ldots, N$. Using the χ^2 procedure find the expression for "a" in terms of the t_i and d_i. Assume that the data have constant errors.

(2) * Find the Gaussian LU decomposition of the matrix

$$\mathbf{A} = \mathbf{LU} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Solve the equation

$$\mathbf{Ax} = \mathbf{y} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

by double back substitution, i.e. solve

$$\mathbf{Lz} = \mathbf{y} \quad \text{and} \quad \mathbf{Ux} = \mathbf{z}.$$

Multiply the matrix **A** by your solution to find **y** in order to show that your answer is correct.

(3) a) Write a code which constructs a matrix and a y-vector (inserted at the end of the matrix) and calls "`elim`" to perform the Gaussian elimination to put it in **LU** form. Remove the statement that constructs the **L** matrix and print out the upper matrix. Replace the statement and print out the matrix with both parts. Use the sample matrix given in Eq. 3 of the text.
b) Construct a subroutine, `SOLVE(A,N,LDIM,NS)`, which returns the answer(s) in the last column(s) of the augmented matrix `A`. Assume that the size of the system is `n` and that there are `ns` input vectors and answers. Use the vector **z** obtained by the Gaussian elimination procedure. Save a copy of the original matrix and multiply the solution vector that you obtain by the matrix and compare it with the original **y**-matrix to prove that you have a correct solution.
c) Write another code `BSUB` which does only the two back substitutions as shown in Eqs. 5.19 starting with the original input vector and the LU matrix which results from the code in part b. Compare your result with the solution in part b.
Use the following matrix system.

$$\begin{pmatrix} 0.546 & 0.447 & 0.242 & 0.194 & 0.795 \\ 0.380 & 0.276 & 0.581 & 0.108 & 0.416 \\ 0.721 & 0.022 & 0.853 & 0.068 & 0.312 \\ 0.151 & 0.759 & 0.186 & 0.597 & 0.757 \\ 0.112 & 0.509 & 0.041 & 0.411 & 0.632 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0.468 \\ 0.695 \\ 0.398 \\ 0.913 \\ 0.483 \end{pmatrix}$$

(4) Using Eq 5.1 find the starting point, initial velocity and acceleration of a moving point given the following data.

t	x
1.0	18.349
1.1	22.021
1.2	25.888
1.3	29.932
1.4	34.380
1.5	38.829
1.6	44.056
1.7	49.422
1.8	54.983
1.9	61.340
2.0	67.582
2.1	74.173
2.2	81.328
2.3	88.368
2.4	96.242

Use Eq. 5.8 and the code developed in problem 2 above of the solution of linear systems. Assume a constant error of 0.2 in the values of x.

(5) * Find a Householder matrix, $P^{(0)}$, which zeros the second and third elements of the column vector

$$\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$$

and apply it to the vector to show that it works. Hint: choose the minus sign in the definition of z.

(6) * Use the Lanczos algorithm, with the starting vector $\mathbf{y}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, to reduce the matrix $\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ to the tridiagonal form, $\mathbf{B} = \begin{pmatrix} a_1 & b_1 & 0 \\ b_1 & a_2 & b_2 \\ 0 & b_2 & a_3 \end{pmatrix}$.
Find the matrix **P** such that the relationship

$$\mathbf{B} = \tilde{\mathbf{P}}\mathbf{A}\mathbf{P}$$

connects them.

Use the following matrix in the solution of the eigenvalue problems.

$$\begin{pmatrix} 3.0895 & 1.0568 & 0.4279 & 0.1046 & -0.1891 \\ 1.0568 & 2.4194 & 0.0424 & -0.4237 & -0.3056 \\ 0.4279 & 0.0424 & 2.8247 & -0.6989 & -0.3561 \\ 0.1046 & -0.4237 & -0.6989 & 2.0324 & -0.8787 \\ -0.1891 & -0.3056 & -0.3561 & -0.8787 & 4.6341 \end{pmatrix} \quad (5.82)$$

(7) Use the power method to find the largest eigenvalue of the matrix 5.82.

(8) Use the inverse power method to find the smallest eigenvalue of the matrix 5.82.

(9) Use the Lanczos method to find all of the eigenvalues of the matrix 5.82.

References

(1) *Computational Methods in Linear Algebra*, R. J. Goult, R. F. Hoskins, J. A. Milner and M. J. Pratt, John Wiley & Sons, (New York-Toronto) 1974, ISBN 0-470-31920-8

(2) *Computational Methods of Linear Algebra*, Granville Sewell, Ellis Horwood (New York) (1990) ISBN 0-13-142811-X

(3) *Computational Methods for Matrix Eigenproblems* A. R. Gourlay and G. A. Watson, John Wiley & sons, (London) (1973) ISBN 0-471-31915-5

(4) *Computational Methods in Classical and Quantum Physics*, Ed. M. B. Hooper, Hemisphere Publishing corporation (Washington DC) (1976) ISBN 0-903012-31-6

(5) *The Symmetric Eigenvalue Problem*, Beresford N. Parlett, Prentice-Hall, Inc., Englewood Cliffs, N. J. 1980, ISBN 0-13-880047-2

(6) *Matrix Calculations*, Gene H. Golub and Charles F. Van Loan, The Johns Hopkins University Press, Baltimore and London, ISBN 0-8018-3739

(7) *Applied Analysis*, Cornelius Lanczos, Prentice Hall, Inc. (1956)